ARBITRAGE COSTS AND NONLINEAR ADJUSTMENTS IN THE G7 STOCK MARKETS

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Fredj JAWADI
University of Evry, France

Georges PRAT
CNRS, France
Purposes

To study the stock price adjustment of the G7 countries toward fundamentals in a nonlinear framework.

To reproduce the phases of under and overvaluation of stock prices toward the equilibrium, using two indicators of Peel and Taylor (2000), never applied on stock markets.
Literature Review

- Barberis et al. (1998): the heterogeneity in investor’s expectations.
Why would stock price adjustment be nonlinear?

**Transaction Costs:** (Dumas, 1992; Anderson, 1997).

Let $z_t = p_t - f_t$ be the actual deviation between the market log-price $p_t$ of a portfolio of equities and its fundamental log-value $f_t$ perceived by all investors.

In the absence of transaction costs and of arbitrage opportunities, any investor can make profits from a stock price deviation. We.

The adjustment process bringing the stock price toward fundamentals is classically continuous and linear with a constant speed of adjustment:

$$
\Delta z_t = -\rho \Delta z_{t-1} + \Phi(L) \Delta z_{t-1} + \nu_t
$$

(1)

where $\Phi(L)$ represents the distributed lag polynomial, $\Delta$ the first difference and $\nu_t$ a white noise.
Why would stock price adjustment be nonlinear?

**Transaction Costs:** (Dumas, 1992; Anderson, 1997).

Nevertheless, arbitrage and transaction are not costless in practice and it now can be seen that the presence of transaction costs and risk aversion reduces arbitrage opportunities. Let \( \tau_t \) represent at time \( t \) the sum of transaction costs (in % of the price) and of a required premium due to the risky character of arbitrage opportunities resulting from uncertainty about the fundamental value perceived. It is worth noting that, in such a configuration, equation (1) is no longer appropriate to reproduce the stock price adjustment dynamics, since it fails to replicate this discontinuity of arbitrages. In this case, the adjustment process takes into account both the no-trade zone and the arbitrage opportunity zone, and can be written as:

\[
\Delta z_t = -\rho \Omega(|z_{t-1}|) z_{t-1} + \Phi(L) \Delta z_{t-1} + \varepsilon_t
\]

where:

- \( \Omega(|z_{t-1}|) = 1 \) if \( |z_{t-1}| > \tau_t \)
- \( \Omega(|z_{t-1}|) = 0 \) if \( |z_{t-1}| \leq \tau_t \)

where \( 0 < \Omega(.) < 1 \) represents a transition function allowing to characterize at each date which regime of the adjustment process holds.

\( \nu_t \) a white noise.
Why would stock price adjustment be nonlinear?

**Transaction Costs:** (Dumas, 1992; Anderson, 1997).

However, stock market transaction costs are heterogeneous. Furthermore, the appreciation of risk associated to arbitrage opportunities is also agent-dependant since risk aversion is an individual preference parameter. The model (2) is no longer appropriate to describe the stock price adjustment. Let $\tau_{j,t}$ be the price of arbitrage associated with the purchase of a unity of portfolio by investor $j$ at time $t$. Let $H(|z_{t-1}|)$ be the cumulative density function of all investors’ expenses, which represent the proportion of equities for which investors expect for time $t$ a benefit due to the price deviation. The adjustment model becomes:

$$\Delta z_t = -\rho \cdot H(|z_{t-1}|) z_{t-1} + \Phi(L) \Delta z_{t-1} + \epsilon_t$$  \hspace{1cm} (3)

where the cumulative density function $H(|z_{t-1}|)$ ranging between 0 and 1, is an exponential function defined as:

$$H(|z_{t-1}|) = H(\tau_t) = 1-\exp\left[-\beta \cdot \tau_t^2\right], \quad \beta > 0 \text{ and } \tau_t \geq 0$$  \hspace{1cm} (4)

where $\beta$ is the transition speed and $\tau_t$ the average of arbitrage cost at time $t$. 
Table 1 - Stock market transaction costs (in % of the amount of the transaction)

<table>
<thead>
<tr>
<th>Transaction costs</th>
<th>Germany</th>
<th>Canada</th>
<th>USA (NYSE)</th>
<th>France</th>
<th>UK</th>
<th>Italy</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct cost</td>
<td>5.51</td>
<td>10.23</td>
<td>5.0</td>
<td>6.58</td>
<td>8.8</td>
<td>10.65</td>
<td>5.9</td>
</tr>
<tr>
<td>Total cost</td>
<td>22.13</td>
<td>23.98</td>
<td>17.51</td>
<td>23.07</td>
<td>24.42</td>
<td>27.78</td>
<td>20.3</td>
</tr>
</tbody>
</table>

Remark: other justifications of nonlinearity

**Heterogeneous Expectations:**

- Investors apply different techniques: chartists, fundamentalists, noise traders (De Grauwe and Grimaldi, 2006).

Fundamental Value modeling:

\[ F_{t+1} = F_t \left(1 + i_{ot} + \Phi_o \right) - E_t(D_{t+1}) \]

where \( i_{ot} \) is the one-period to maturity risk-free rate. \( D_{t+1} \) is the dividend distributed during the period \([t, t+1]\).

\[ D_t = \left(\alpha_0 + \alpha_1 D_{t-1} + \cdots + \alpha_p D_{t-p}\right) + \left(\beta_0 + \beta_1 D_{t-1} + \cdots + \beta_p D_{t-p}\right) \times \Omega \left(D_{t-d}, \gamma, c\right) + \varepsilon_t \]
STECM for stock price deviations:

\[
\Delta z_t = k + \rho_1 z_{t-1} \times [1 - \Omega(\gamma, z_{t-\delta})] + \rho_2 z_{t-1} \times \Omega(\gamma, z_{t-\delta}) + \sum_{i=1}^{\nu} \phi_i \Delta z_{t-i} + \mu_t
\]  

(10)

where \(\rho_1\) and \(\rho_2\) are respectively the adjustment coefficients in the first and second regime, \(z_{t-1}\) is the lagged error-correction term, \(z_{t-\delta}\) is the transition variable, \(\phi_i\) are the AR parameters and \(\mu_t \rightarrow N(0, \sigma_\mu^2)\) is an error term.
**Restricted STECM:**

Following Peel and Taylor (2000), we consider three hypotheses leading to a restricted specification of the STECM which have never been considered for stock markets:

\[
\begin{align*}
H_0^a & : k' = c = 0, \\
H_0^b & : \rho_1 + \rho_2 = -1 / H_0^a, \\
H_0^c & : \rho_1 = 0 \quad \text{s.t.} \quad H_0^a \quad \text{and} \quad H_0^b
\end{align*}
\]  

\[\Delta z_t = - z_{t-1} \times \Omega \left( \gamma, z_{t-d} \right) + \sum_{i=1}^{p} \phi_i \Delta z_{t-i} + \mu_t \]  

(11)
Augmented Restricted STECM:

\[ \Delta z_t = -z_{t-1} \times \Omega(y, z_{t-d}) + \sum_{i=1}^{p} \phi_i \Delta z_{t-i} + \sum_{j=0}^{p'} \alpha_j \Delta z_{t-j}^{USA} + \sum_{j=0}^{p'} \theta_j \Delta i_{0,t-j} + \sum_{j=0}^{p'} \theta'_{j} \Delta q_{t-j} + \mu_t \]

Gauging under- and overvaluation phases and adjustment strengths

\[ \Pi(z_t) = 100 \times \Omega(z_t) \times \text{sign}(z_t), \quad \text{sign}(z_t) = \frac{z_t}{|z_t|}, \quad -100 \leq \Pi(z_t) \leq 100 \quad (14) \]

\[ \Psi(z_t) = 1 - \Omega(z_{t-d}), \quad 0 \leq \Psi(z_t) \leq 1 \quad (15) \]
DATA and Preliminarily Results:

Monthly data: Stock indexes, Interest rates and industrial production Series of the G7 countries.

Period: December 1969 – March 2005

Sources: International Monetary Fund’s International Financial Statistics and MSCI databases.

Preliminarily tests: All stock indexes are I(1). Besides, stock returns are asymmetric and leptokurtic.
Estimating Fundamental Value (FV) under the REH:

We compute an initial FV, noted $F_0$: We swept it in the interval $[P_0 - 50\%, P_0 + 50\%]$. We also suppose that the initial constant risk premium $\Phi_0$ belongs to the interval $[0\%, 8\%]$. Then, we determine the optimal value of $F_0$, so that the retained value of $F_0$ and $\Phi_0$ are:

$$Q = \sum_{t=1}^{n=T} (p_t - f_t)^2$$

where $p_t$ and $f_t$ are respectively the log-values of prices and fundamental values.
• Estimating Fundamental Values

Then, we replace the expected dividends by the STARM estimation and we got the following FV estimations:
• **STECM modeling for stock price deviations**

**Specification Tests (1):**

We determine \( p \). Then, we test the linearity by LM tests of Luukkonen *et al.* (1988) for several values of \( d \): \( 1 \leq d \leq 12 \).

<table>
<thead>
<tr>
<th>Delay</th>
<th>Germany</th>
<th>Canada</th>
<th>USA</th>
<th>France</th>
<th>UK</th>
<th>Italy</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \hat{d} )</td>
<td>10</td>
<td>2</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>p-value</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
</tr>
</tbody>
</table>

*Note: \( p \) is the number of lags in the change of the deviation. \( \hat{d} \) is the optimal number of lags in the transition variable \( z_{t-d} \).*
• **STECM modeling for stock price deviations**

**Specification Tests (2): Selecting the transition function**

<table>
<thead>
<tr>
<th>Countries</th>
<th>Delay parameter</th>
<th>$H_{03}$</th>
<th>$H_{02}$</th>
<th>$H_{01}$</th>
<th>$H_{0L}$</th>
<th>$H_{0E}$</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>10</td>
<td>0.09</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.001</td>
<td>ESTECM</td>
</tr>
<tr>
<td>Canada</td>
<td>2</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.008</td>
<td>0.00</td>
<td>ESTECM</td>
</tr>
<tr>
<td>The USA</td>
<td>6</td>
<td>0.0009</td>
<td>0.00</td>
<td>0.001</td>
<td>0.003</td>
<td>0.00</td>
<td>ESTECM</td>
</tr>
<tr>
<td>France</td>
<td>2</td>
<td>0.15</td>
<td>0.008</td>
<td>0.04</td>
<td>0.002</td>
<td>0.00</td>
<td>ESTECM</td>
</tr>
<tr>
<td>The UK</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>ESTECM or LSTECM</td>
</tr>
<tr>
<td>Italy</td>
<td>6</td>
<td>0.21</td>
<td>0.002</td>
<td>0.54</td>
<td>0.007</td>
<td>0.00</td>
<td>ESTECM</td>
</tr>
<tr>
<td>Japan</td>
<td>10</td>
<td>0.24</td>
<td>0.004</td>
<td>0.001</td>
<td>0.00</td>
<td>0.00</td>
<td>ESTECM or LSTECM</td>
</tr>
</tbody>
</table>

*Table 3: Selecting the transition function $\Omega(.)$*
• **STECM modeling for stock price deviations**

**Specification Tests (3):**

We test the non-restricted STECM against the restricted STECM by a likelihood ratio test. All restrictions are accepted for all stock indexes.

<table>
<thead>
<tr>
<th>Countries</th>
<th>Germany</th>
<th>Canada</th>
<th>USA</th>
<th>France</th>
<th>UK</th>
<th>Italy</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR(^a)</td>
<td>0.8</td>
<td>0.79</td>
<td>0.85</td>
<td>0.58</td>
<td>0.12</td>
<td>0.79</td>
<td>0.28</td>
</tr>
<tr>
<td>LR(^b)</td>
<td>0.89</td>
<td>0.93</td>
<td>0.98</td>
<td>0.82</td>
<td>0.09</td>
<td>0.77</td>
<td>0.11</td>
</tr>
<tr>
<td>LR(^c)</td>
<td>0.93</td>
<td>0.74</td>
<td>0.97</td>
<td>0.90</td>
<td>0.08</td>
<td>0.67</td>
<td>0.80</td>
</tr>
</tbody>
</table>

*Note: the table gives the p-values issued from the LR test.*
<table>
<thead>
<tr>
<th>p</th>
<th>1</th>
<th>3</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{d}_r$</td>
<td>10</td>
<td>2</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>$\hat{\gamma}$</td>
<td>0.62 (3.8)$^*$</td>
<td>0.1 (4.4)$^*$</td>
<td>0.57 (3.6)$^*$</td>
<td>8.53 (3.29)$^*$</td>
<td>0.64 (1.63)$^{**}$</td>
<td>9.94 (2.7)$^*$</td>
<td>7.65 (2.18)$^{**}$</td>
</tr>
<tr>
<td>$\hat{\phi}_1$</td>
<td>-0.06 (-1.75)$^{**}$</td>
<td>-0.08 (-1.63)$^{**}$</td>
<td>-0.03 (-1.69)$^{**}$</td>
<td>0.06 (2.1)$^*$</td>
<td>-0.02 (-0.44)</td>
<td>0.14 (2.9)$^*$</td>
<td>-0.02 (-1.63)$^{**}$</td>
</tr>
<tr>
<td>$\hat{\phi}_2$</td>
<td>-</td>
<td>-0.02 (-1.1)</td>
<td>-</td>
<td>-</td>
<td>-0.46 (-9.7)$^*$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\phi}_3$</td>
<td>-</td>
<td>0.17 (5.4)$^*$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\alpha}_0$</td>
<td>0.16 (3.07)$^*$</td>
<td>0.68 (16.1)$^*$</td>
<td>-</td>
<td>0.44 (7.9)$^*$</td>
<td>1.08 (21.7)$^*$</td>
<td>0.98 (13.1)$^*$</td>
<td>0.06 (1.2)</td>
</tr>
<tr>
<td>$\hat{\alpha}_1$</td>
<td>0.12 (2.4)$^*$</td>
<td>0.16 (2.9)$^*$</td>
<td>-</td>
<td>-</td>
<td>-0.05 (-0.9)</td>
<td>0.38 (5.08)$^*$</td>
<td>0.35 (6.07)$^*$</td>
</tr>
<tr>
<td>$\hat{\alpha}_2$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.37 (6.2)$^*$</td>
<td>0.42 (5.8)$^*$</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\alpha}_0'$</td>
<td>0.19 (3.6)$^*$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\alpha}_0''$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.9 (20.4)$^*$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\beta}_0$</td>
<td>-</td>
<td>-</td>
<td>0.18 (3.9)$^*$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\theta}_0$</td>
<td>-0.007 (-1.73)$^{**}$</td>
<td>-0.01 (-4.2)$^*$</td>
<td>-0.03 (-6.06)</td>
<td>-0.02 (-5.8)$^*$</td>
<td>-0.005 (-1.8)$^{**}$</td>
<td>-0.06 (-10.3)$^*$</td>
<td>-0.01 (-2.3)$^*$</td>
</tr>
<tr>
<td>$\hat{\theta}_1$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.34 (1.98)$^*$</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\gamma} \times \sigma_z^2$</td>
<td>0.07 (0.8)</td>
<td>0.006 (1.8)$^{**}$</td>
<td>0.08 (1.8)$^{**}$</td>
<td>1.2</td>
<td>0.04</td>
<td>1.3</td>
<td>1.1</td>
</tr>
<tr>
<td>ADF (p)</td>
<td>-13.9$^*$ (p = 0)</td>
<td>-14.3$^*$ (p = 0)</td>
<td>-14.8$^*$ (p = 0)</td>
<td>-14.6$^*$ (p = 0)</td>
<td>-20.3$^*$ (p = 0)</td>
<td>-14.6$^*$ (p = 0)</td>
<td>-14.07$^*$ (p = 0)</td>
</tr>
<tr>
<td>DW</td>
<td>1.97</td>
<td>2.04</td>
<td>2.02</td>
<td>2.03</td>
<td>2.01</td>
<td>2.0</td>
<td>2.02</td>
</tr>
<tr>
<td>Q(4)</td>
<td>0.12</td>
<td>0.6</td>
<td>2.07</td>
<td>1.5</td>
<td>0.95</td>
<td>4.6</td>
<td>2.2</td>
</tr>
<tr>
<td>Q(12)</td>
<td>5.31</td>
<td>29.2</td>
<td>9.34</td>
<td>13.07</td>
<td>14.2</td>
<td>15.5</td>
<td>6.7</td>
</tr>
<tr>
<td>ARCH (q)</td>
<td>5.06$^*$ (q = 1)</td>
<td>10.8$^*$ (q = 1)</td>
<td>14.3$^*$ (q = 1)</td>
<td>0.55$^*$ (q = 1)</td>
<td>17.7$^*$ (q = 1)</td>
<td>7.9$^*$ (q = 1)</td>
<td>18.8$^*$ (q = 2)</td>
</tr>
<tr>
<td>N</td>
<td>18</td>
<td>47</td>
<td>30</td>
<td>45</td>
<td>27</td>
<td>25</td>
<td>28</td>
</tr>
</tbody>
</table>
• **STECM estimations:**

  • Strong evidence of nonlinear stock price adjustment.

  • Significant persistence and slowness in stock price adjustment.

  • Significant contagion effects between stock markets and dependence to the US stock market.

  • Negative relationships between stock indexes and interest rates for all indexes.

  • Validation of estimation results by misspecification tests of Eithreim and Teräsvirta (1996).
• **Estimated transition functions:**

![Graphs of USA and France estimated transition functions](image)
• Gauging under and Overvaluation Phases:
• Measuring Adjustment Strengths
• Comments:

• Long durations of under- and overvaluations of the MSCI-G7 stock indexes.

• An average adjustment delay from price to fundamentals of about 5 months.

• The stock price mean-reversion speed is time-varying, asymmetrical and nonlinear. This speed is more important for higher deviations and it is higher during the periods of crises (1973, 1979, 1982).
Main conclusions and contributions

- An on/off threshold ECM with two regimes to reproduce stock price adjustment.

- In the first regime, stock price deviations approach a random walk, while they approach a white noise process in the outer regimes.

- Stock prices may deviate and be away for long time, but they are non-linearly mean-reverting when deviations exceed some threshold given by transaction costs.
Thank you!!!