Estimating the S&P Fundamental Value Using STAR Models

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MOTIVATION (1)

1- Important stock market movement because of: Reduction of transaction costs, Increase of the transaction volumes and the number of investors.

Examples:

- In 1987, the Dow Jones displayed a rise of 250% in relation to its low level of 1982. It has more than tripled in five years to clear the rode of 11000 points in 1999.

- The S&P recorded in January 2003 a fall of 40%. How can explain these movements?
Recently, the US companies recorded an increasing of their production, profits and earnings, returns, implying thus an increase in dividend yields and a stimulation of the stock prices (i.e. Shiller (2000)).
How, can we explain this important development of American stock market after 2000?
Interrogations:

- Are stock price evolutions justified by fundamentals?
- Do changes in the stock prices reflect those occurring in the fundamentals?
- How can we estimate the fundamental value (FV)?
- Which discount rate is appropriate?
- How can we represent the expected future cash flows?
Hypotheses

✓ $H_1$: We define the FV as the discounted sum of the expected future cash flows while retaining the Dividend Discount Model (DDM) and applying Campbell and Shiller Linearization.

✓ $H_2$: The CSL implies that the Dividend Yield Ratio is stationary and that dividends rise with a memorised rate.

✓ $H_3$: After linearizing the stock return around stationary variables, we got the following FV expression:
This relation defines the FV as an increasing function of expected future dividends and a decreasing function of discount rate.

Then, while computing the relation \((\rho \ p^{f}_{i+1} - p^{f}_i)\), we obtained the following relation:

\[
\rho p^{f}_{i+1} - p^{f}_i = (1-\rho) \lambda^* + (g-r^*) + \rho d_{i+1} - d_t + E_t (r_{i+1}) - E_t (\Delta d_{i+1})
\]

Therefore, the fundamental price is given by the following recurrent relation:

\[
p^{f}_{i+1} = \frac{1}{\rho} p^{f}_i + \frac{1-\rho}{\rho} \lambda^* + \frac{g-r^*}{\rho} + d_{i+1} - \frac{1}{\rho} d_t + \frac{1}{\rho} E_t (r_{i+1}) - \frac{1}{\rho} E_t (\Delta d_{i+1})
\]
Fundamental Value Modeling

In practice, this empirical formulation of the fundamental price is not directly measured. We need, on the one hand, to define an initial value for the fundamental price $p_0^f$ to start the recurrent relation. On the hand, we have to identify an expectation processes for future expected dividends and discount rate. Thus, we retained the rational expectations hypothesis and we defined the expected sets of the above relations as follows:

$$E_t (r_{t+1}) = r_{t+1} + \varepsilon_{t+1}$$

(13)

$$E_t (\Delta d_{t+1}) = \Delta d_{t+1} + \varepsilon_{t+1}$$

(14)

Future sets would be generated while using STAR models while under the above assumption $\varepsilon_{t+1}$ must to have the properties of a white noise process. Using STAR models is justified by the asymmetry, the persistence and the structural breaks induced by the presence of transaction costs, the behavioural heterogeneity, the different dividends policies and the heterogeneous beliefs (e.g. Drifill and Sola (1998)).

Otherwise, different initial value $p_0^f$ is proposed. First, we retained $p_0^f = p_0$ to generate a fundamental value serie. Then, other values are tested and the optimal one is the value that minimizes the following statistic:

$$Q = \sum_{i=0}^{\infty} (p_i - p_i^f)^2.$$
**Empirical Results**

*Data and Preliminary Tests*

- Monthly American Stock Index: **S&P500**.
- **Source**: Shiller Database.
- **Notice**: All Data are real.

*Unit Root Tests*

- ADF, PP and KPSS Tests checked the conditions of CSL while showing that $P_t$ and $D_t$ are I(1) and the $DY$ is I(0).
Smooth Transition Autoregressive Models

1. STAR were introduced by Teräsvirta and Anderson (1992) and Teräsvirta (1994).

2. STAR modeling is defined in two stages: Specification (AR modeling, Linearity Tests) Estimation (NLS Method).
**STAR Specification**

- **Step 1: Specification of linear model**
  - The number of delays is chosen on the basis of information criterias (AIC, BIC), $K_{max}$ test and Ljung-Box tests.

  We retained an AR(6) and AR(3) respectively for the returns and dividends.

- **Step 2: Linearity Tests**
  - We tested linearity hypothesis against its alternative of nonlinearity of STAR type. Thus, we used four tests: (LM$_1$, LM$_2$, LM$_3$, and LM$_4$). These tests were developed by Van Dijk, Teräsvirta and Franses (2002).

  - Linearity is strongly rejected for both series and we retained $d = 4$ and $d = 1$ respectively for the returns and dividends.
\[ \hat{r}_t = \begin{pmatrix} 0.04 & 0.22 \ r_{t-1} & 0.17 \ r_{t-2} & 0.51 \ r_{t-3} & 6.68 \ r_{t-4} & 1.16 \ r_{t-5} & 0.37 \ r_{t-6} \\ 0.72 & 1.85 & -1.96 & -1.95 & 0.65 & 2.88 & -1.99 \end{pmatrix} \times \exp \left\{ -\frac{0.493}{(1.81)} \left( \frac{r_{t-4}}{(7.82)} + 0.005 \right)^2 \right\} + \begin{pmatrix} 0.03 & 0.13 \ r_{t-1} & 0.006 \ r_{t-2} & 0.02 \ r_{t-3} & 0.04 \ r_{t-4} & 0.08 \ r_{t-5} & 0.04 \ r_{t-6} \\ 10.9 & 12.7 & -0.23 & -1.77 & 1.74 & 2.83 & 1.74 \end{pmatrix} \times 1 - \exp \left\{ -\frac{0.493}{(1.81)} \left( \frac{r_{t-4}}{(7.82)} + 0.005 \right)^2 \right\} \]

\( \text{DW} = 1.98, \ ADF = -28.15, \ ARCH(4) = 18.73, \ JB = 4.79 \) and \( \frac{\sigma_{\text{ESTAR}}}{\sigma_{\text{AR}}} = 0.7. \)

\[ \Delta \hat{d}_t = \begin{pmatrix} 0.001 & 0.54 \ \Delta d_{t-1} & 0.11 \ \Delta d_{t-2} & 0.07 \ \Delta d_{t-3} \\ 1.29 & 2.86 & 1.79 & -1.74 \end{pmatrix} \times \exp \left\{ -\frac{9.2 \times 10^{-4}}{(1.69)} \left( \Delta d_{t-1} - 0.0032 \right)^2 \right\} + \begin{pmatrix} -0.0002 & 0.39 \ \Delta d_{t-1} & 0.16 \ \Delta d_{t-2} & 0.11 \ \Delta d_{t-3} \\ -0.41 & 13.5 & 4.77 & 3.04 \end{pmatrix} \times 1 - \exp \left\{ -\frac{9.2 \times 10^{-4}}{(1.69)} \left( \Delta d_{t-1} - 0.0032 \right)^2 \right\} \]

\( \text{DW} = 2.01, \ ADF = -15.56, \ ARCH(4) = 16.67, \ JB = 8.79 \) and \( \frac{\sigma_{\text{ESTAR}}}{\sigma_{\text{AR}}} = 0.66. \)
STAR Estimation

Our results are in line with those of Drifill and Sola (1998) and Sarantis (2001) implying the superiority of the ESTAR models and indicating that the S&P return and dividend adjustment dynamics are rather nonlinear.

Finally, in order to propose a new estimation for the S&P fundamental value, we retained the deterministic estimation of STAR models of $r_t$ and $\Delta d_t$. Then, we expressed estimation in $(t+1)$ to deduce $E_t (r_{t+1})$ and $E_t (\Delta d_{t+1})$ and we estimated the S&P fundamental price given the equation (12). But in order to start the recurrent relation, we supposed that $p^f_0 = p_0$. Estimated fundamental price and S&P index are reproduced in the following graph.
As in Manzan (2003), we showed that changes in fundamental factors are not large enough to explain changes in observed price. Indeed, the American index was over-valuated in the beginning of the period. It fluctuates however around its fundamental value until 1990, but it knew a considerable run-up after 1990, indicating the absence of mean reversion in American stock prices in this period.
VI. **Conclusion and possible extensions**

The main contribution of this paper is to develop a new empirical measure of S&P fundamental value while extending the DDM by introducing the nonlinearities in estimating the fundamental value. We showed, on the one hand, that nonlinear models are more appropriate than linear model to reproduce returns dynamics and expected future dividends evolution. On the other hand, we found that price is nonlinearly mean-reverting. Indeed, price could deviate and spend most of the time away from fundamental at short term, but while transaction costs and the extent of its deviations from fundamental are increasing, a strong evidence of mean-reverting in American stock price is found.
Thank You